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A Case Study of Employing A Single Server Nonpreemptive Priority Queuing Model at ATM Machine

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ABSTRACT
This paper discusses a case study of employing a single server nonpreemptivepriorityqueuing model [1]at ATM machine which originally operates on M/M/1 model. In this study we have taken two priority classes of people in following order:-
.Priority class 1- woman
.Priority class 2- man
Sometimea long queue is formed at ATMmachine (single server)but the bank management don’t have enough money to invest on installing new ATM machine.In this situation we want to apply single server nonpreemptive priority queuing model.The security guard at the ATM will divide the customers in two category and arrange the customers in the above said priority order Thuspriority class 1 people willreceive theatm service ahead of priority class 2 people.This will reduce the waiting time of priority class 1 people. Of course by doing this the waiting time of priority class 2will increase. This is ok as long as the increment in waiting time of priority class2 people is reasonable and within the tolerable limitof priority class 2people.This will be true when percentage of priority class 1 people is relatively less as compared to priority class 2 people To know the attitude and tolerable limit of priority class 2 people towards the single server non preemtive priority model a sample survey has been done on the incomingpriority class 2 population at the atm machine.Against this background, the queuing process is employed with emphasis to Poisson distribution to assess the waiting time. The data for this study was collected from primary source and is limited to ATM service point of state bank of India located at Ramesh chowk, Aurangabad, bihar, India.. The assistance of three colleague was sought in collecting the data. The Interarrival time and service time data was collected during busy working hours (i.e. 10.30am to 4:00pm) during the first 60 days. A sample survey was done to know the attitude and tolerable limit of priority class 2people towards the single server nonpreemptive priority model. The result of sample survey was enthusiastic and favoured the implementation of thesingle server nonpreemitive priority queuing model at atm machine.
Keywords: Automatic Teller Machine, M/M/1 Queuing models, Nonpreemptive Priorities Queuing Model, Poisson Distribution, Queuing Process, Sample Survey, Tolerable Limit

I. Introduction

Although there has been significant reforms in recent times all in an effort to maximize profit, reduce cost and satisfy customers optimally in the most generally acceptable international standard. Despite these entire sterling efforts one phenomenon remains inevitable: queue. It is a common practice to see a very long waiting line of customers to be serviced either at the Automated Teller Machine (ATM) or within the banking hall. Though similar waiting lines are seen in places like; busstop, fast food restaurants, clinics and hospitals, traffic light, supermarket, etc. but long waiting line in the banking sector is worrisome being the public’s most important units.

[2]Queue is a general phenomenon in everyday life. Queues are formed when customers (human or not) demanding service have to wait because their number exceeds the number of servers available; or the facility doesn’t work efficiently or takes more than the time prescribed to service a customer. Some customers wait when the total number of customers requiring service exceeds the number of service facilities, some servicefacilities stand idle when the total number of service facilities exceedsthe number of customers requiring service. [3] defines queue as simply a waiting line, while[4] put it in similar way as a waiting line by two important elements: the population source of customer from which they can draw and the service system. The population of customer could be finite or infinite.

II. Queuing Theory [1]

2.1 Basic Structure Of Queuing Models

2.1.1 The Basic Queuing Process

The basic process assumed by most queuing models is the following. Customers requiring service are generated over time by an input source. These customers enter the queuing system and join a queue. At certain times, a member of the queue is selected for service by some rule known as the queue discipline. The required service is then performed for thecustomer by the servicemechanism, after which the customer leaves the queuing system. This process is depicted in Fig. 1 Many alternative assumptions can be made about the various elements of the queuing process; they are discussed next.

2.1.2 Input Source (Calling Population)

One characteristic of the input source is its size. The size is the total number of customers that might require service from time to time, i.e., the total number of distinct potential customers. This population from which arrivals come is referred to as the calling population. The size may be assumed to be either infinite or finite (so that the input source also is said to be either unlimited or limited).The statistical pattern by which customers are generated over time must also be specified. The common assumption is that they are generated according to a Poisson process; i.e., the number of customers generated until any specific time has a Poisson distribution. This case is the one where arrivals to the queuing system occur randomly but at a certain fixed mean rate, regardless of how many customers already are there (so the size of the input source is infinite). An equivalent assumption is that the probability distribution of the time between consecutive arrivals is an exponential distribution. The time between consecutive arrivals is referred to as the interarrival time. Any unusual assumptions about the behaviour of arriving customers must also be specified. One example is balking, where the customer refuses to enter the system and is lost if the queue is too long.

2.1.3 Queue

The queue is where customers wait before being served. A queue is characterized by the maximum permissible number of customers that it can contain. Queues are called infinite or finite, according to whether this number is infinite or finite. The assumption of an infinite queue is the standard one.

2.1.4 Queue Discipline

The queue discipline refers to the order in which members of the queue are selected for service. For example, it may be first-come-first-served, random, according to some priority procedure, or some other order. First-come-first-served usually is assumed by queuing models, unless it is stated otherwise.

2.1.5 Service Mechanism

The service mechanism consists of one or more service facilities, each of which contains one or more parallel service channels, called servers. If there is more than one service facility, the customer may receive service from a sequence of these (service channels in series). At a given facility, the customer enters one of the parallel service channels and is completely serviced by that server. A queuing model must specify the arrangement of the facilities and the number of servers (parallel channels) at each one. Most elementary models assume one service facility with either one server or a finite number of servers. The time elapsed from the commencement of service to its completion for a customer at a service facility is referred to as the service time (or holding time).The service-time distribution that is most frequently assumed isexponential distribution.

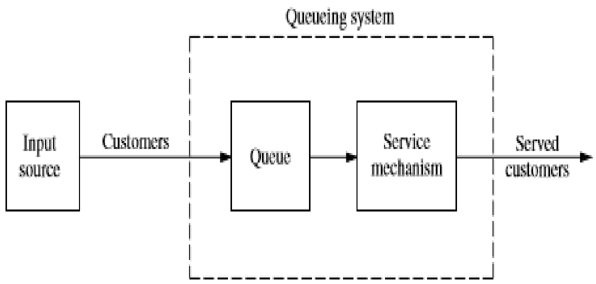

Figure-1 the basic queuing process

2.2 Little's Theorem[1]

Theorem describes the relationship between throughput rate (i.e. arrival and service rate), cycle time and work in process (i.e. number of customers/jobs in the system). The theorem states that the expected number of customers (N) for a system in steady state can be determined using the following equation:

$L =$ (1)

2.3 Atm Model (M/M/1 Queuing Model)[1]

M/M/1 queuing model means that the arrival and service time are exponentially distributed (Poisson process). For the analysis of the ATM M/M/1queuing model, the following variable will be investigated:

λ=Thecustomer's mean arrival rate

μ =The customer's mean service rate

.W: Average waiting time in the system:

$$W = \frac{1}{\mu - \lambda} \qquad (2)$$

. Wq: Average waiting time in queue:

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\lambda(\mu - \lambda)} \qquad (3)$$

2.4 Non preempitive Priority Model[1]

. W_K : Steady-state expected waiting time in the system (including service time) for a member of priority class k:

$$W_K = \frac{1}{A * B_{K-1} * B_K} + \frac{1}{\mu}, \text{ for K=1, 2...N} \qquad (4)$$

Where

N=number of priority classes

$B_O = 1$

$B_K = 1 - \frac{\sum_{i=1}^{K} \lambda_i}{s\mu}$ s=no of servers

μ=mean service rate per server

λ_i =mean arrival rate for priority class i

$r = \sum_{i=1}^{N} \lambda_i$ (This results assume that $\sum_{i=1}^{k} \lambda_i < s\mu$ so that the priority class K can reach a steady state condition)

. W_{Kq} : Steady-state expected waiting time in the queue for a member of priority class k:

$$W_{Kq} = W_K - \frac{1}{\mu} \qquad (5)$$

III. Calculations

In this section first of all the goodness of fit test of the interarrival time data and service time has been done. Then arrival rate and service rate of the entire incoming population at atm machine is determined. This arrival rate and service rate is used to calculate the arrival rate and service rate of priority class 1 and priority class 2 people. Then waiting time in system/queue is calculated using M/M/1 model and then using single server non preemptive priority model. Finally a sample survey has been done to determine the attitude of priority class2 people towards the implementation of single server non preemptive priority model and also to determine the tolerable limit of priority class 2 people.

3.1The Inter Arrival Time Data For The Entire Incoming Population At ATM Machine That Was Collected During The First 60 Days

3.1.1 Frequencies For The Inter Arrival Time

Table-1Frequencies For Interarrival Time For The Entire Incoming Population At ATM Machine

X	Y
0-1	1382
1-2	1352
2-3	949
3-4	641
4-5	452
5-6	408
6-7	209
7-8	191
8-9	180
9-10	181
10-11	103
11-12	60
12-13	30

At X=0-1 implies that 1382 times, customers arrived at an inter arrival time between 0 to 1minute.

3.1.2 Histogram Of Interarrival Time Data Of The Entire Incoming Population

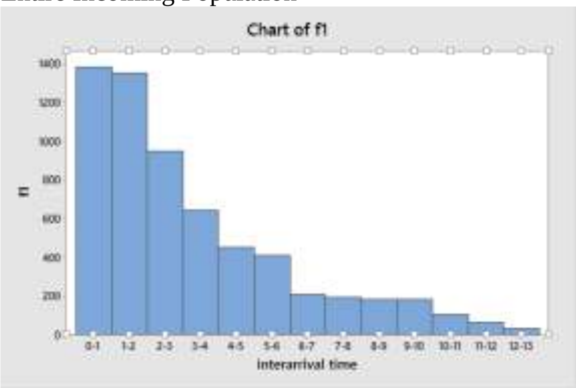

Figure-2histogram of interarrival time

3.2 The Service Time DataFor The Entire Incoming Population At ATM Machine That Was Collected During The First 60 Days

3.2.1 Frequencies For The Service Time

Table-2Frequencies For Service Time For The Entire Incoming Population At ATM Machine

M	N
0-1	2179
1-2	2100
2-3	1004
3-4	525
4-5	172
5-6	97
6-7	50
7-8	11

At M=0-1 implies that 2179 times, customers arrived at an inter arrival time between 0 to 1minute

3.2.2 Histogram Of Service Time Data Of The Entire Incoming Population At ATM Machine

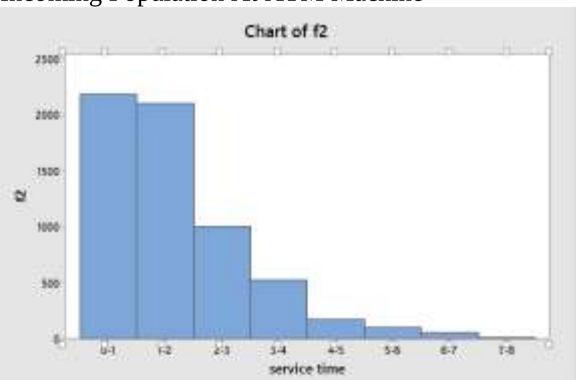

Figure-3 histogram of service time

3.3 Checking The Goodness Of Fit Of The Entire Interarrival Time And Service Time Data For The Incoming Population Using χ^2 Method [5]

The histogram shows us the shape of sample data and from it we can roughly infer that it resembles an exponential curve. It is good approximation for starting.However, this graph only tells us about the data from this specific example. To make any inferences about the larger population to increase the usefulness of these data, we check the fitness of our data to exponential curve. We make the use of Minitab for this.A value of p>0.05 indicates a good fit. A low p-value (< 0.05) indicates that the data don't follow the distribution.In our case the value of p is 0.812 for Interarrival time data and 0.072 for service timedata (where p is the probability corresponding to the calculated value of χ^2 for the given degree of freedom).Thus it indicates that the given set of data is quiet a fair representative of the entire population.

3.4Arrival Rate And Service RateOf Entire Incoming PopulationAt ATM Machine [6]

Since we have checked the goodness of fit of data, we are now in a position to calculate arrival rate and service rate. And these calculated values will be used subsequently.

Interarrival time of the entire incoming population at atm machine= $\frac{1}{\lambda} = \frac{\sum_{i=1}^{13} X_i * Y_i}{\sum_{i=1}^{10} Y_i} = \frac{19800}{6138} = 3.2258$

Arrival rate per hour= $\lambda = \frac{60}{3.2258} = 18.6$

Service time of the entire incoming population at atm machine= $\frac{1}{\mu} = \frac{\sum_{i=1}^{8} M_i * N_i}{\sum_{i=1}^{8} N_i} = \frac{10302}{6138} = 1.6784$

Service rate per hour= $\mu = \frac{60}{1.6784} = 35.7483$

3.5CompositionOf Entire Incoming Population At ATM Machine

From observation during the first 60days, following composition of incoming population at atm was found:-

.Number of priority class 1 people-614.

Therefore % of priority class 1 people $= \frac{614}{6138} * 100 = 10.0033\% \approx 10\%$

.Number of priority class 2 people-5524

Therefore % of priority class 2 people=100%-10%=90%

3.6 Arrival Rate And Service Rate Of Priority Class 1 And Priority Class 2 People

Arrival rate of priority class 1 people= $\lambda_1 = 10\% of\, \lambda = 10\% of\, 18.6 = 1.86$ (Since 10 % of people are priority class 1 people)

Arrival rate of priority class 2 people= $\lambda_2 = 90\% of\, \lambda = 90\% of\, 18.6 = 16.74$ (Since 90 % of people are priority class 2 people)

The mean service rate of all category of people essentially remain same.

3.7Waiting time in the system and in the queue

3.7.1 Using M/M/1 Queuing Model

Equation (1)-(2) is used to calculate waiting time.

Waiting time in system= $W = 0.0583hr = 3.498\min = 209.88\sec$

Waiting time in queue= $W_q = 0.0303hr = 1.818\min = 109.08\sec$

3.7.2 Using Single Server Nonpreemptive priority model

Equation (3)-(5) is used to calculate waiting time

For a member of priority class 1:-

Waiting time in system of priority class 1 people= $W_1 = 0.0433hr = 2.598\min = 155.88\sec$

Waiting time in queue of priority class 1 people= $W_{1q} = 0.0154hr = 0.924\min = 55.44\sec$

For a member of priority class 2:-

Waiting time in system of priority class 2 people= $W_2 = 0.0600hr = 3.6\min = 216\sec$

Waiting time in queue of priority class 2 people= $W_{2q} = 0.0320hr = 1.92\min = 115.2\sec$

Table 3The steady state waiting time in the queue calculated above is shown in table for the M/M/1 queuing model and Single Server Nonpreemptive Priorities Modelfor comparison purpose:-

Parameter	M/M/1 (FIFO)	Priority class 1	Priority class 2
Waiting time in the system (sec)	209.88 (W)	155.88(W_1)	216 (W_2)
Waiting time in the queue (sec)	109.08 (W_q)	55.44(W_{1q})	115.2 (W_{2q})

Table 4Decrease in waiting time of priority class 1 people

Decrease in waiting time in system of priority class 1 people	$W - W_1 =$ $209.88\sec - 155.88\sec =$ $54\sec$
Decrease in waiting time in queue of priority class 1 people	$W_q - W_{1q} =$ $109.08\sec - 55.44\sec =$ $53.64\sec$

Table 5Increase in the waiting time of priority class 2 people

Increase in the waiting time in the system of priority class 2 people	W_2-W = 216-209.88=6.12 sec
Increase in the waiting time in the queue of priority class 2 people	W_{2q}-W_q =115.2-109.08=6.1 sec

3.8 Sample survey

Sample survey was done to determine the attitude and tolerable limit of priority class2 people towards the single server non preemptive priority model

3.8.1 Tolerable Limit

We define the tolerable limit as follows**:-**

Tolerable limit of priority class 2 people = how much more time than their normal waiting time priority class 2 people is willing to spend in queue as a result of implementing single server non preemptive priority model.

3.8.2 Survey[7][8]

A sample survey was done at the same atm machine to determine the attitude and tolerable limit of priority class 2 people.The survey was carried out through a total of three hundred and eighty five (385) priority class 2 people. Copies of Questionnaire was given to three hundred and eighty five (385) Stochastically Selected priority class 2 people .Population of this Study Is Infinite and Arrived Rate Was Random and Exponentially Distributed. The Sample Size of 385 was Derived Using the Freund and Williams Formula for an infinite population:

Samplesize=

$$N = \frac{Z^2 * p * q}{d^2} = \frac{1.96^2 * 0.5 * 0.5}{0.05^2} = 384.16 \cong 385$$

Where:

Z=Statistical certainty usually chosen at 95% confidence level= 1.96

P=Percentage picking a choice = 0.5

q = 1-p =0.5

d=Precision desired =5%= 0.05

3.8.3 Results of sample survey

Out of 385 priority class 2 people 374 people were in favour of single server non preemptive priority model and had tolerable limit between 10-40 seconds. Only 11 people said they don't want this model to be implemented.

IV. Conclusion

From survey we see that the tolerable limit of priority class 2 people is between 10-40seconds.Fromthe Table 3we see that the waiting time in the system and in the queuesing M/M/1 model is 209.88 sec and 109.08 sec respectively.Also we see from Table 3that the waiting time in the system and in queuesing single server non preemptive priority model for priority class 1 people is 155.88sec and 55.44 sec respectively. From Table 4 we see thatafter applying single server non preemtive priority model, priority class 1 peoplenow have to spend 54 second 53.64 sec less respectively in the system and in queue.From Table 5 we see that after applying single server non preemtive priority model there is very slight increase in the waiting time in the system (6.12sec) and inqueue (6.12sec) for priority class 2 people and these incrementare well below the interval between 10sec-40sec which is the tolerable limit. On account of above discussion single server nonpreemitive priority queuing modelcould be applied in our case without affecting the customer demand, revenue and profit of the bank. This case study will act as a reference for implementing single server non preemptive priority models in atm machine

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